Correspondence

An Equivalent Circuit of the Internal Cavity Reflex Klystron-Amplifier

The experimental results of microwave amplification by the reflex klystron 2K25 have been reported.[1-4] An equivalent circuit of the 2K25 amplifier was developed and checked with the experimental results.

The fundamental circuit arrangement of the 2K25 internal cavity reflex klystron amplifier is shown in Fig. 1. The tube is mounted in the middle of the waveguide. The center conductor of the coaxial output line is extended and terminated by a variable reactor. Microwave signals are fed into the left opening of the waveguide and taken out of the right.

An equivalent circuit of the 2K25 reflex klystron amplifier is shown in Fig. 2. The letters $A, B, C, \cdots$, in Fig. 1 correspond to the letters $A, B, C, \cdots$, in Fig. 2. In this case, losses in the cavity resonator and transmission lines are neglected. In Fig. 2,

$l_{1,2,3}$ = line-lengths of the coaxial lines,
$\lambda_{1,2,3}$ = wavelengths on the lines,
$Z_{01,2,3}$ = characteristic impedances of the lines
$Z_{g1,2}$ = driving point impedances across the terminals FH, Z_{g1} for the input side and Z_{g2} for the output side,
X_s = a reactance of the variable reactor.

The equivalent parallel resistance R_{sh} and the parallel reactance X_{sh} of the external circuit impedance across the grids of the reflex klystron 2K25 are given by elementary analysis of the equivalent circuit shown in Fig. 2, as follows:

$$R_{sh} = Z^2/R_{CD}(\omega M)^2, \quad (1)$$

$$X_{sh} = -Z^2/[\omega C_P Z^2 + (X_{CD} + \omega L_s)(\omega M)^2 - \omega^2 L_s L_P - X_{CD}\omega L_P - \omega L_P R_{CD}^2] \quad (2)$$

where

$$Z^2 \equiv \{(\omega M)^2 - \omega^2 L_s L_P - X_{CD}\omega L_P\}^2 + \omega^2 L_P^2 R_{CD}^2. \quad (3)$$

Here, R_{CD} and X_{CD} are equivalent series resistance and reactance of the right-hand side of the network across the terminals CD in Fig. 2, and are given by either analysis or Smith Chart. Apparently these are functions of X_s. Thus, the values of R_{sh} and X_{sh} can be adjusted by changing X_s, the reactance of the variable reactor.

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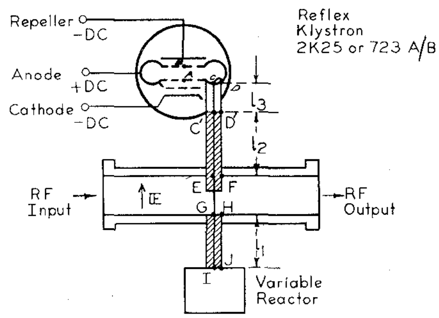

Fig. 1—Fundamental circuit of internal cavity reflex klystron amplifier.

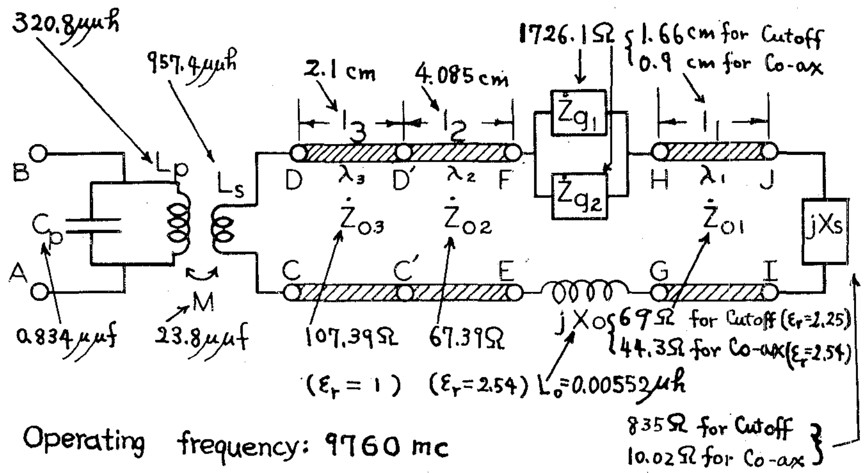

Fig. 2—Equivalent circuit of internal cavity reflex klystron amplifier.

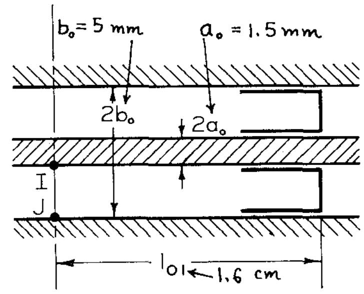

Fig. 3—Coaxial variable reactor.

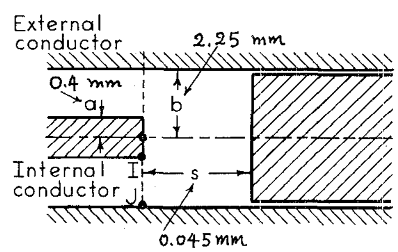

Fig. 4—Cutoff variable reactor.

The value of X_s for the coaxial variable reactor shown in Fig. 3 is given simply by

$$X_s = Z_{00} \tan(2\pi l_{01}/\lambda), \quad (4)$$

where Z_{00} is a characteristic impedance of the coaxial line. On the other hand, the value of X_s for the cutoff variable reactor, which was newly designed for this amplifier circuit, shown in Fig. 4, is given by the following equation:

$$X_s = \frac{\dfrac{\sqrt{\mu/\epsilon}}{16\pi ab\sigma s}}{\lambda} \cdot \frac{\{I_0(\sigma b) - I_0(\sigma a)\}\{aK_1(\sigma a) - bK_1(\sigma b)\} - \{K_0(\sigma b) - K_0(\sigma a)\}\{bI_1(\sigma b) - aI_1(\sigma a)\}}{I_1(\sigma b)K_1(\sigma a) - I_1(\sigma a)K_1(\sigma b)} \quad (5)$$

TABLE I

OUTPUT IMPEDANCE ACROSS THE GRID OF THE REFLEX KLYSTRON

Reactor	R_{sh}	X_{sh}	$R_{sh} \cos \phi$
Cutoff	$2.99 \times 10^5\ \Omega$	$-3.91 \times 10^3\ \Omega$	$3.91 \times 10^3\ \Omega$
Coaxial	$2.75 \times 10^5\ \Omega$	$-19.01 \times 10^3\ \Omega$	$1.89 \times 10^3\ \Omega$

TABLE II

GAIN MEASURED AND CALCULATED

Reactor	Gain (db) Measured	Anode Voltage V_0	Repeller $-V_r$	I_0 (ma)	N	β	Gain (db) Calculated
Cutoff	23.2	280	88	7.7	10.75	0 525	22 71
Coaxial	28.0	304	268	2.5	6.75	0 55	29.1

Note: $\beta = \sin \dfrac{3170\, dg}{2\lambda\sqrt{V_0}} \Big/ \dfrac{3170\, dg}{2\lambda\sqrt{V_0}}, \quad N = 4fl \Big/ \sqrt{\dfrac{m}{2e}\, V_0/(V_0+V_r)}$

$l = 3.44 \times 10^{-3}$ m,
Gap-distance $dg = 0.61 \times 10^{-3}$ m.

where I_s' and K_s' are the modified Bessel functions and

$$\sigma \equiv \sqrt{\left(\frac{\pi}{4s}\right)^2 - \left(\frac{2\pi}{\lambda}\right)^2}. \tag{6}$$

This cutoff reactor gives the same positive reactance for extremely small distance of the shorting plunger setting in comparison with the coaxial reactor. The cutoff reactor is mechanically simpler than the coaxial type.

The numerical circuit constants used to obtain the values of R_{sh} and X_{sh} are shown in Figs. 2–4. The dimensions are based on measurement of the actual circuit used. The constants of the klystron cavity C_P, L_P, L_s, and M were calculated by Fujisawa's method.[5,6] The impedance Z_{g1} and Z_{g2} in Fig. 2 were calculated by Tanaka's method.[7] The reactance of the antenna, X_0, was calculated assuming it was a uniform cylindrical conductor. The computed results of R_{sh} and X_{sh} are given in Table I, where $\cos \phi$ is the power factor of the circuit. These values were checked by the experiment in the following way.

The gain of the reflex klystron amplifier can be calculated by the following equation[8] based on the regenerative action of the electron beam.

$$A = \frac{V_0}{V_0 - I_0\beta^2 R_{sh}\pi N \cos\phi} \tag{7}$$

where

A = gain,
V_0 = anode voltage,
I_0 = effective electron beam current,
β = beam coupling coefficient,
N = number of electron transit time cycles in the repeller space.

The calculated results are listed in Table II with some measured data such as anode and repeller voltage, effective current, and dimensions of the tube. As shown in this table, the calculated gains are in good agreement with the measured gains.

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Some Comments on the Method of Kyhl*

The purpose of this letter is not to criticize the philosophy of Kyhl's[1] method nor even to engage in a debate as to whether his method is, or is not, more useful than ours.[2,3] We find his proposals both interesting and meaningful. First, we would like to call attention to two errors in his letter and then to show the close relation of his method to ours.

Let us consider his method. He proposes that a "Γ" $= 1/\Gamma$ be used when $|\Gamma| > 1$. This, he claims, will produce his "double SMITH-HTIMS chart." This is in error because, if $|\Gamma| > 1$, the original point in the extended Standard Smith chart[3] is without the unit circle, and the inversion type operation (with or without the minus sign) within the unit circle; hence, the "HTIMS" part of the chart will coincide with the regular Smith chart.

To obtain Kyhl's "double SMITH-HTIMS chart," we suggest the following construction. An inversion in the unit circle, followed by a symmetry[4] (reflection) with respect to the line $\Gamma = 1$. This can be written analytically as "Γ" $= -\Gamma'^* + 2$, where $\Gamma'^* = 1/\Gamma^*$.

We next consider the statement that our method is not analytic. By this statement, we presume that Kyhl means that the modified β and the modified β^{-1} transformations, or the Darboux transformation[5] as shown in footnote 2, is presented in its natural graphical form. The following is the transformation in analytic form:

$$[\Gamma_1 = 1/\Gamma^*, \quad \ln\,[O\Gamma', AB] = 2 \ln\,[O\Gamma_1, AB],$$

$$\Gamma' = \frac{2\Gamma_1}{1 + |\Gamma_1|^2} = \frac{2\Gamma}{1 + |\Gamma|^2}$$

where A and B are the two points on the unit circle, intersected by the straight line, connecting the center (O) and the inverse point (Γ_1). The brackets indicate the cross ratio of the four points in question. The final analytic form shows the invariance of the final point with respect to inversion in the unit circle, which was proven geometrically.[2] It is noted that the above equation is essentially the same as that stated by Deschamps[6] for the β transformation.

In comparing our results with Kyhl's, we shall use his method with our modification. It is conceded that there are other ways to correct the proposed method. The first step of inversion is the same in both procedures. The difference occurs in the second step. Our transformation (which is of course the Deschamps'[7] β and β^{-1} transformation) is best considered as finding the non-Euclidean bisector of a line segment.[6,8] The second step in Kyhl's procedure, the reflection, is an involution.[9] The inversion[9] (also an involution), followed by the reflection, is a graphical way of performing a nonloxodromic bilinear transformation. The unit circle is the isometric circle of the equivalent bilinear transformation, which is "Γ" $= (2\Gamma - 1)/\Gamma$. This result also could have been obtained analytically, instead of using the geometrical interpretation. In summary, it is noted that Kyhl's and our results are quite similar in form, both analytically and geometrically, but differ mostly in the final presentation.

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